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Opposite-current flows in gas–liquid boundary layers — I. Velocity distribution

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Abstract

A theoretical analysis of gas–liquid counter-current flow in laminar boundary layers with flat phase boundary based on similarity variables method has been done. The obtained numerical results for the velocity distribution in both phases are compared with analogous results from asymptotic theory and experimental data. The dissipation energy in boundary layer is determined and the results corresponding to counter-current and co-current flows are compared. The comparison shows significant differences in dissipation energy values in gaseous phase. © 2000 Elsevier Science Ltd. All rights reserved.

Keywords: Opposite-current flow; Gas–liquid system; Boundary layer; Velocity distribution

1. Introduction

The chemical technologies based on opposite-current flows in gas–liquid systems are widely distributed in practice. The theoretical analysis of such flows [1] demonstrates that there is a possibility to obtain asymptotic solutions for gas–liquid systems which are in agreement with the experimental data, obtained from thermo-anemometrical measurements of the velocity distribution in boundary layers. The exactness of the proposed asymptotic method [1] requires to be confirmed by numerical experiments.

The experience in exact solution of the problem by means of numerical simulation [2] shows that it is a non-classical problem of mathematical physics which is not sufficiently discussed in literature. A prototype of such problem is the parabolic boundary value problem with changing direction of time [3,4]. It was shown [2] that this non-classical problem can be described as consisting of several classical problems. The same approach will be used in the present work for determination of velocity distribution in gas–liquid opposite-current flows with flat phase boundary.

2. Mathematical model

The mathematical description of the opposite-current flows (Fig. 1) in approximation of boundary layer theory has the following form:

$$u_i\frac{\partial u_i}{\partial x}+v_i\frac{\partial u_i}{\partial y}=\upsilon_i\frac{\partial^2 u_i}{\partial y^2},\qquad \frac{\partial u_i}{\partial x}+\frac{\partial v_i}{\partial y}=0;$$

$$x=0,\quad y\geq 0,\quad u_1=u_1^{\infty};\qquad x=l,\quad y\leq 0,\quad u_2=-u_2^{\infty};$$

$$y\to\infty,\quad 0\leq x\leq l,\quad u_1=u_1^{\infty};$$

$$y\to-\infty,\quad 0\leq x\leq l,\quad u_2=-u_2^{\infty};$$

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Nomenclature

u velocity in x direction (m/s)
v velocity y direction (m/s)
x coordinate (m)
y coordinate (m)

Greek symbols
μ dynamic viscosity (N/m s deg)
ρ density (kg/m^3)
υ kinematic viscosity (m^2/s)

Subscripts
1 for gas
2 for liquid

Superscript
$*$ for co-current flow

$$y=0,\quad 0<x<l,\quad u_1=u_2,$$
$$\mu_1\frac{\partial u_1}{\partial y}=\mu_2\frac{\partial u_2}{\partial y},\quad v_1=v_2=0. \tag{1}$$

The problem (1) can be presented in dimensionless form using two different coordinate systems for the two phases, so that the flow in each phase is oriented to the longitudinal coordinate, and the following dimensionless variables and parameters are introduced:

$$x=lX_1=l-lX_2,\qquad y=\delta_1Y_1=-\delta_2Y_2,$$

$$u_1=u_1^\infty U_1,\qquad v_1=u_1^\infty\frac{\delta_1}{l}V_1,$$

$$u_2=-u_2^\infty U_2,\qquad v_2=-u_2^\infty\frac{\delta_2}{l}V_2,$$

$$\delta_i=\sqrt{\frac{\upsilon_i l}{u_i^\infty}},\quad i=1,2,$$
$$\theta_1=\frac{u_2^\infty}{u_1^\infty},\quad \theta_2=\left(\frac{\rho_1\mu_1}{\rho_2\mu_2}\right)^{1/2}\left(\frac{u_1^\infty}{u_2^\infty}\right)^{3/2}. \tag{2}$$

In the new coordinate systems, the model of opposite-current flows has the following form:

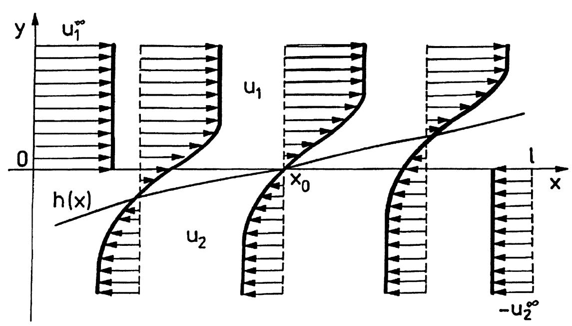

Fig. 1. Counter-current flow.

$$U_i\frac{\partial U_i}{\partial X_i}+V_i\frac{\partial U_i}{\partial Y_i}=\frac{\partial^2U_i}{\partial Y_i^2},\qquad \frac{\partial U_i}{\partial X_i}+\frac{\partial V_i}{\partial Y_i}=0;$$

$$X_i=0,\quad U_i=1;\qquad Y_i\to\infty,\quad U_i=1;$$

$$Y_1=Y_2=0,\quad U_1=-\theta_1U_2,\quad \theta_2\frac{\partial U_1}{\partial Y_1}=\frac{\partial U_2}{\partial Y_2},$$
$$V_i=0;\ i=1,2. \tag{3}$$

3. Method of solution

The problem (3) cannot be solved directly, because the velocities $U_i\,(i=1,2)$ change their directions in domains $0\le X_i\le 1,\ 0\le Y_i<\infty,\ (i=1,2)$. This non-classical problem of mathematical physics can be presented [3] as a classical one after the introduction of the following similarity variables:

$$U_i=f_i',\quad V_i=\frac{1}{2\sqrt{X_i}}\left(\eta_i f_i'-f_i\right),\quad f_i=f_i(\eta_i),$$
$$\eta_i=\frac{Y_i}{\sqrt{X_i}}. \tag{4}$$

Substitution of Eq. (4) into Eq. (3) leads to:

$$2f_i'''+f_if_i''=0,$$

$$f_i(0)=0,\quad f_i(\infty)=1,\quad i=1,2,$$

$$f_1'(0)=-\theta_1f_2'(0),\quad \theta_2\sqrt{\frac{X_2}{X_1}}f_1''(0)=f_2''(0),$$
$$X_1+X_2=1. \tag{5}$$

It is obvious from Eq. (5) that the problem (3) has no solution in similarity variables. However, the problem (5) can be solved after the introduction of new parameter $\bar{\theta}_2$ for each $X_1\in(0,1)$:

Table 1
Numerical results of the boundary conditions

x_1	$\bar{\theta}_2$	a	b	$f'_1(6)$	$f'_2(6)$
$\theta_1 = 0.1$, $\theta_2 = 0.152$					
1.00	0.00000	0.099895	0.3265000	0.998970	0.998950
0.95	0.03487	−0.097863	0.3268130	0.998971	0.998970
0.90	0.05067	−0.096930	0.3269220	0.998973	0.998950
0.85	0.06385	−0.096150	0.3270100	0.998971	0.998977
0.833	0.06806	−0.095900	0.3270400	0.998974	0.998980
0.80	0.07600	−0.095410	0.3271000	0.998982	0.998830
0.75	0.08776	−0.094718	0.3271720	0.998972	0.998960
0.70	0.09951	−0.094000	0.3272520	0.998970	0.998860
0.65	0.11153	−0.093282	0.3273320	0.998974	0.998975
0.60	0.12410	−0.092510	0.3274150	0.998970	0.998910
0.55	0.13748	−0.091690	0.3275050	0.998973	0.998929
0.50	0.15200	−0.090800	0.3275980	0.998970	0.998984
0.45	0.16804	−0.089800	0.3277050	0.998972	0.998963
0.40	0.18620	−0.088650	0.3278240	0.998971	0.998857
0.35	0.20714	−0.087330	0.3279600	0.998973	0.998916
0.30	0.23220	−0.085730	0.3281200	0.998972	0.998971
0.25	0.26327	−0.083710	0.3283180	0.998972	0.998976
0.20	0.30400	−0.080998	0.3285750	0.998973	0.998890
0.167	0.33950	−0.078598	0.3287940	0.998973	0.998967
0.15	0.36183	−0.077058	0.3289300	0.998972	0.998972
0.10	0.45600	−0.070300	0.3294910	0.998970	0.998942
0.05	0.66255	−0.053540	0.3306320	0.998972	0.998919

$$\bar{\theta}_2 = \theta_2 \sqrt{\frac{1 - X_1}{X_1}}$$

$$\bar{\theta}_2 = \theta_2 \sqrt{\frac{1 - X_1}{X_1}}, \tag{6}$$

i.e. the problem has local similarity solution. In this way, the problem (5) is substituted by several separate problems for each $X_1 \in (0, 1)$.

The solutions of these separate problems can be obtained after the introduction of the function F:

$$F(a, b) = \int_6^7 (f'_1 - 1)^2 \, d\eta_1 + \int_6^7 (f'_2 - 1)^2 \, d\eta_2,$$

$$a = f'_1(0), \quad b = f''_1(0). \tag{7}$$

The solution of Eq. (5) for each $X_1 \in (0, 1)$ is obtained after finding the minimum of the function $F(a, b)$, where at each step of minimization procedure the boundary problem has to be solved:

$$2f'''_i + f_i f''_i = 0, \quad f_i(0) = 0, \quad i = 1, 2,$$

$$f'_1(0) = a, \quad f'_2(0) = -\frac{a}{\theta_1}, \quad f''_1(0) = b,$$
$$f''_2(0) = \bar{\theta}_2 b. \tag{8}$$

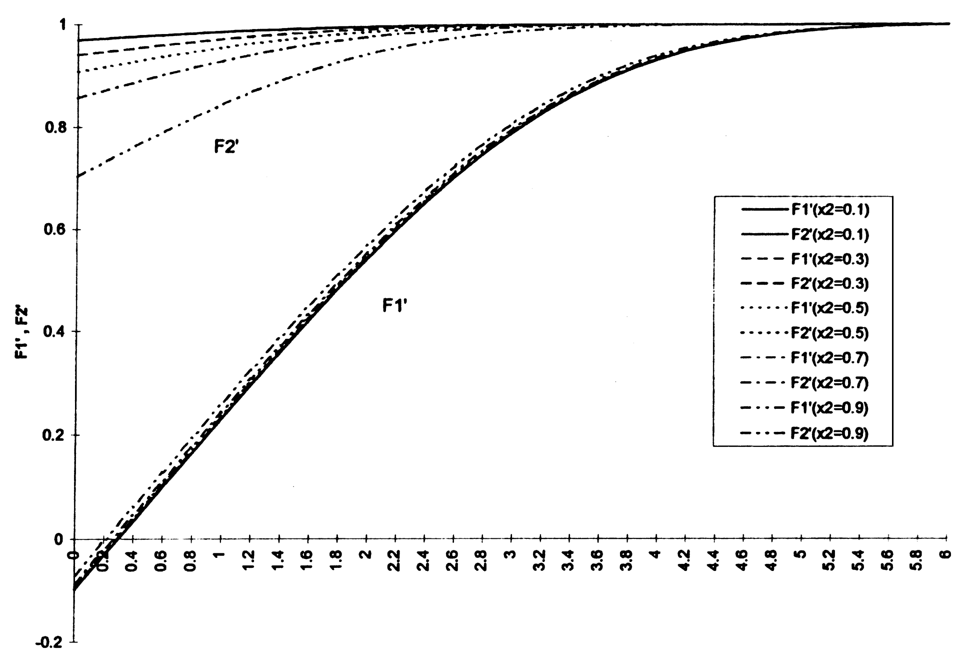

Fig. 2. Numerical results of the velocity distribution.

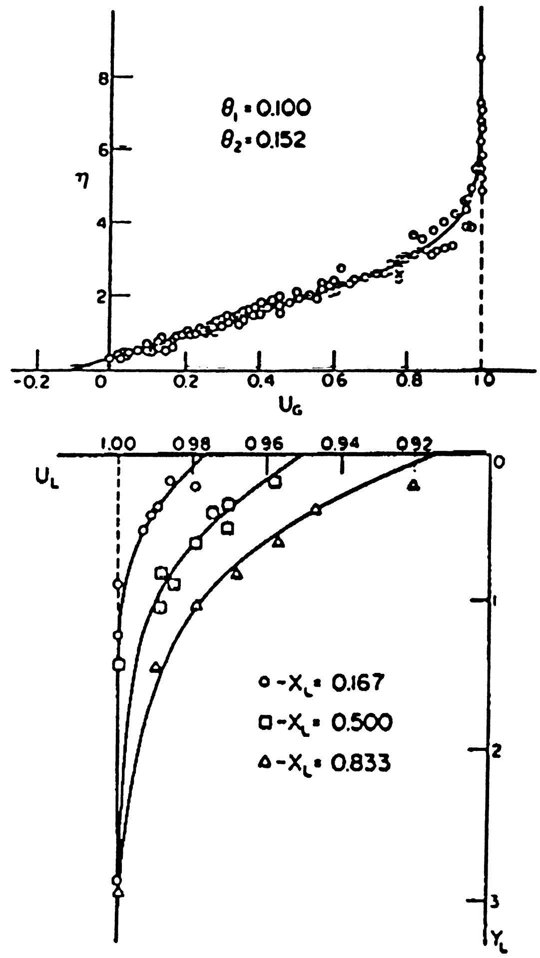

Fig. 3. Theoretical (asymptotic solution) and experimental velocity profiles in counter-current flow.

4. Numerical results

The problem (8) was solved numerically for opposite-current gas (1) and liquid (2) flows for the following parameters' values $\theta_1 = 0.1$ and $\theta_2 = 0.152$. In accordance with the requirement for minimum of $F(a, b)$ in Eq. (7), the boundary conditions a, b and $F(a, b)$ were determined. The results obtained for $f_1'(6)$ and $f_2'(6)$ are shown in Table 1. Taking into account that boundary layer theory gives $f'(6) = 0.99897$ [5], the obtained results are characterized with sufficient precision at determination of a, b.

The velocity distribution in the gaseous and liquid phases are presented in Fig. 2. The boundary conditions in Eq. (5) show that the velocity at the interphase boundary becomes zero when $X_1 = X_1^0$:

$$f_i'(0) = 0, \quad i = 1, 2, \tag{9}$$

Table 2
Comparison between asymptotic and numerical theory (velocity distribution)

$\theta_1 = 0.1$, $\theta_2 = 0.152$			
η_2	Y_2	$U_2(X_2, Y_2)$	f_2'
		$X_2 = 0.167$	
0.00	0.0	0.907672	0.959000
1.22	0.5	0.945604	0.983229
2.45	1.0	0.983392	0.995563
3.67	1.5	0.997511	0.998583
4.89	2.0	0.999821	0.998962
		$X_2 = 0.5$	
0.00	0.0	0.910754	0.908000
0.71	0.5	0.941204	0.942042
1.41	1.0	0.966199	0.968796
2.12	1.5	0.983722	0.985933
2.83	2.0	0.993640	0.994414
3.54	2.5	0.998019	0.997703
4.24	3.0	0.999513	0.998693
		$X_2 = 0.833$	
0.00	0.0	0.883033	0.785989
0.55	0.5	0.927292	0.846172
1.10	1.0	0.955745	0.899398
1.64	1.5	0.974572	0.940078
2.19	2.0	0.986680	0.968257
2.74	2.5	0.993754	0.984835
3.29	3.0	0.997420	0.993261
3.83	3.5	0.999070	0.996915

therefore:

$$\bar{\theta}_2 = \theta_2 \sqrt{\frac{1 - X_1^0}{X_1^0}} = 1, \tag{10}$$

because in order to fulfil the conditions $f_i'(\infty) = 1$, it is necessary that

$$f_1''(0) = f_2''(0) = 0.33205. \tag{11}$$

It follows directly from Eq. (10) that at $\theta_2 = 0.152$ for the point where the phase velocity changes its direction, X_1^0 is:

$$X_1^0 = 0.02252. \tag{12}$$

The results from the asymptotic theory [1] present the velocity change at the boundary layer $U_2(X_2, Y_2)$ and at the phase boundary $U_2(X_2, 0)$:

$$U_2(X_2, Y_2) = 1 - \theta_2 \frac{0.33205}{\sqrt{\pi}} \int_0^{X_2} \frac{\exp\left[-Y_2^2/4(X_2 - \xi)\right]}{\sqrt{(X_2 - \xi)(1 - \xi)}} \, d\xi,$$

$$U_2(X_2, 0) = 1 - \theta_2 \frac{0.33205}{\sqrt{\pi}} \ln \frac{1 + \sqrt{X_2}}{1 - \sqrt{X_2}}. \tag{13}$$

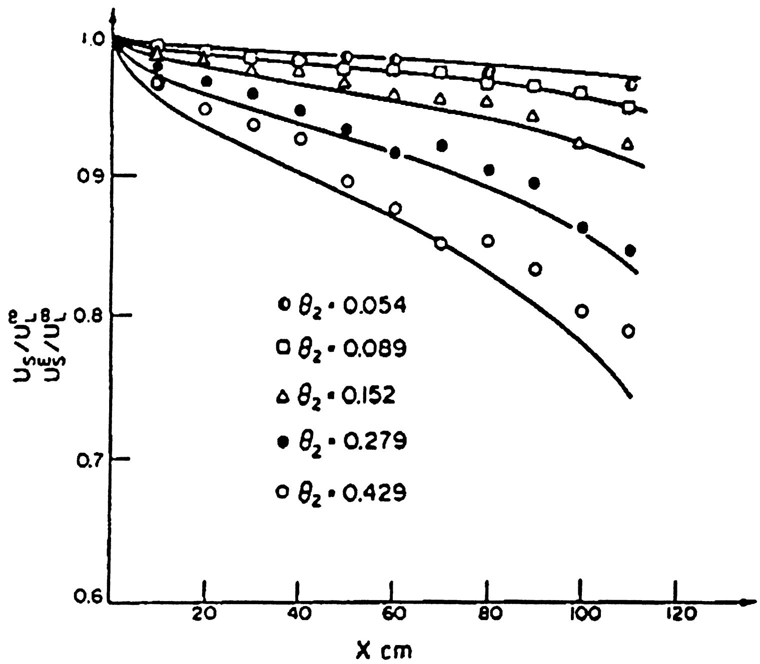

Fig. 4. Theoretical (asymptotic solution) and experimental relationships of surface velocities of the length X at various θ_2 in counter-current flow.

They are compared [1] with the experimental data in Figs. 3 and 4 ($\eta = \eta_1$, $U_G = f_1'$, $X_L = X_2$, $Y_L = Y_2$, $U_L = U_2$, $U_S = U_2(X_2, 0)$, $U_L^\infty = U_2^\infty$, $U_S^E =$ experimental data) and with the directly calculated results in Tables 2 and 3, where $f_2'(\eta_2)$ is a numerical solution of the problem (5). The comparison shows a good agreement between the asymptotic theory (13) and numerical solution, taking into account that accuracy of the asymptotic theory is about 10–15%. From Tables 2 and 3 and Figs. 3 and 4, a good agreement between the results of physical experiments and numerical simulation is seen.

Table 3
Comparison between asymptotic and numerical theory (interphase velocity)

$\theta_1 = 0.1$, $\theta_2 = 0.152$		
X_2	$U_2(X_2, Y_2)$	f_2'
0.1	0.98135	0.96930
0.2	0.97259	0.95410
0.3	0.96496	0.94000
0.4	0.95754	0.92510
0.5	0.94980	0.90800
0.6	0.94124	0.88650
0.7	0.93109	0.85730
0.8	0.91778	0.80998
0.9	0.89643	0.70300

The obtained results show that there is a line, where the velocity changes its direction in gaseous phase (Fig. 5).

5. Energy dissipation

The energy dissipated in the laminar boundary layer [6,7] is described for both phases by the equation:

$$e_i = \mu_i \int_0^l \int_0^{(-1)^{i+1}} \left(\frac{\partial u_i}{\partial y}\right)^2 \mathrm{d}x\,\mathrm{d}y, \quad i = 1, 2. \tag{14}$$

Using dimensionless variables (2), the problem (14) takes the following form:

$$E_i = -\int_0^1 \int_0^\infty \left(\frac{\partial U_i}{\partial Y_i}\right)^2 \mathrm{d}Y_i\,\mathrm{d}X_i, \quad i = 1, 2, \tag{15}$$

where

$$E_i = \frac{e_i\sqrt{\left(U_i^\infty l\right)/\upsilon_i}}{\upsilon_i \rho_i U_i^{\infty^2}}, \quad i = 1, 2. \tag{16}$$

The introduction of similarity variables leads to:

$$E_i = \int_0^1 \frac{1}{\sqrt{X_i}}\left[\int_0^\infty \left(f_i''^2\right)\mathrm{d}\eta_i\right]\mathrm{d}X_i, \quad i = 1, 2. \tag{17}$$

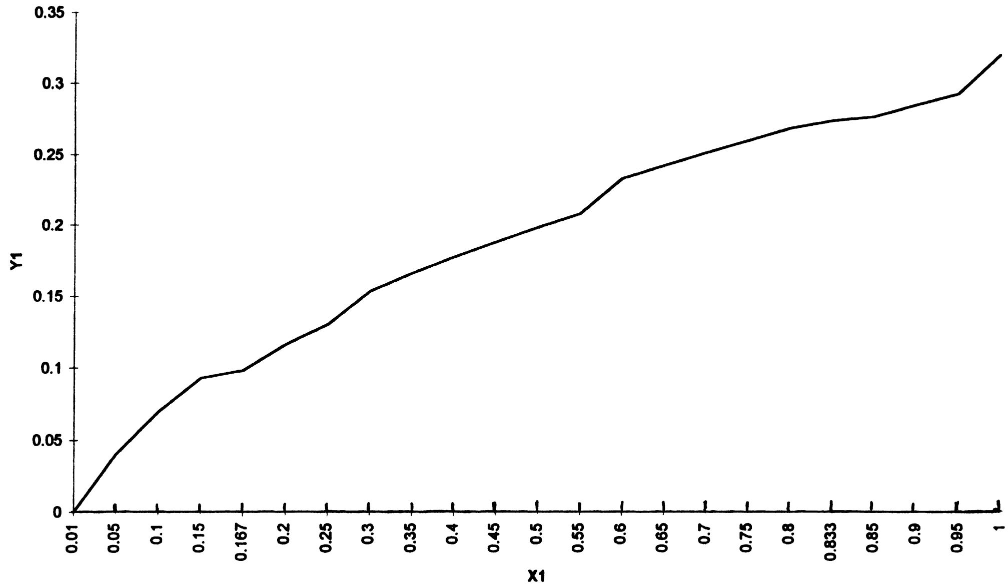

Fig. 5. Zero velocity line.

Table 4
Comparison between the energy dissipation for co-current and counter-current two-phase flows

	Gas	Liquid
$\theta_1 = -0.1$, $\bar{\theta}_2 = \theta_2 = 0.152$	$E_1^* = 0.458334$	$E_2^* = 0.0064370$
$\theta_1 = 0.1$, $\theta_2 = 0.152$	$E_1 = 0.52505$	$E_2 = 0.0132823$

In case of co-current flows, f''^*_i does not depend on X_i and for dissipation energy the following is obtained:

$$E_i^* = 2\int_0^\infty (f''^*_i)^2 \, d\eta_i, \quad i = 1, 2, \tag{18}$$

where f_i^* $(i = 1, 2)$ is the solution of Eq. (8) at boundary conditions for co-current flows:

$$\theta_1^* = -0.1, \quad \theta_2^* = \theta_2 = 0.152, \quad f'^*_1(0) = 0.0908, \quad f''^*_1(0) = 0.32765. \tag{19}$$

In Table 4, the dimensionless energy dissipation $E_i\,(i = 1, 2)$ in the boundary layer is shown for the case of gas–liquid opposite-current flows. It is compared with values obtained for co-current flows $E_i^*\,(i = 1, 2)$.

These results show that the energy dissipation $E_i^*\,(i = 1, 2)$ for gaseous phase in case of co-current flows is lower than that in case of opposite-current flows $E_i\,(i = 1, 2)$, while in the second (liquid) phase there is no significant change.

6. Conclusion

The obtained results allow determination of the velocity distribution in opposite-current flows in gas–liquid boundary layers. They open the sociability for a theoretical analysis of the heat and mass transfer kinetics under these conditions. The comparison between opposite-current and co-current flows shows significant differences in dissipation energy values in the gaseous phase.

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